MATH 15200

Calculus

University of Chicago

January 29, 2020

Still Section 4.2: Disk/Washer Method

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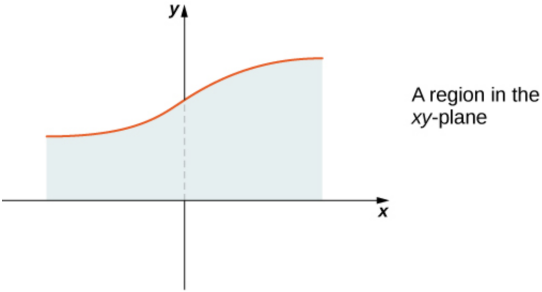

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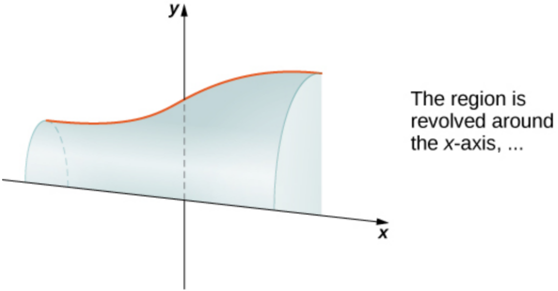

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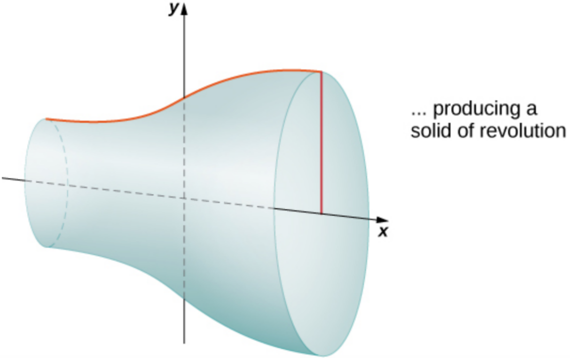
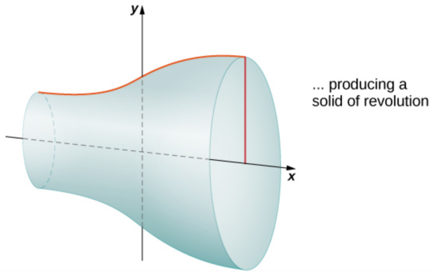




We want to find the volume of this solid of revoluion.

The cross-section at x of this solid is a circle of radius $f(x)$.

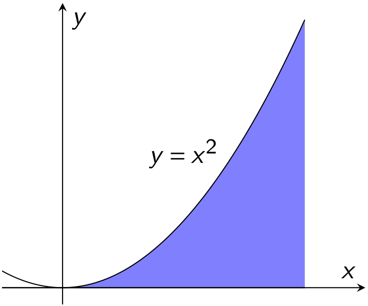
Therefore, its area is $A(x) = \pi[f(x)]^2$.

And its volume is

$$\mathrm{Vol}(S) = \pi \int_a^b [f(x)]^2 \, dx.$$

Example 1

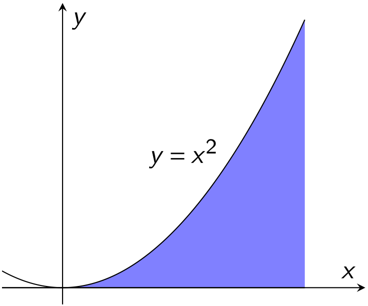
Consider the following region:



Revolve this region around the x-axis and calculate the volume of the resulting solid S.

Example 1

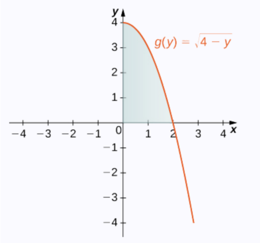
Consider the following region:



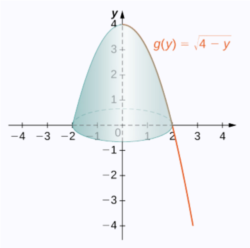
Revolve this region around the x-axis and calculate the volume of the resulting solid S.

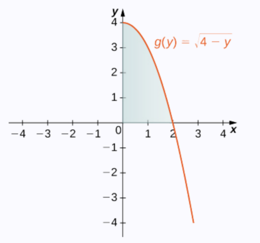
$$\mathrm{Vol}(S) = \pi \int_0^4 (x^2)^2 \, dx = \frac{\pi}{5} x^5 \Big|_0^4 = \frac{1024\pi}{5}.$$

We can also revolve around the y-axis:



Becomes...



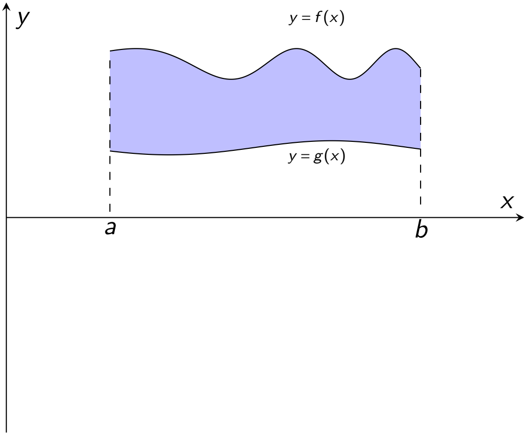


To calculate the volume of this region, we slice with horizontal planes. The cross-sections are circles with radius $\sqrt{4-y}$. Therefore, the cross-sectional area is $A(y) = \pi(4-y)$. And the volume is

$$\text{Vol}(S) \;=\; \pi \int_0^4 (4-y)\,dy \;=\; 8\pi.$$

Washer Method

If we have a region that looks like We revolve around the x-axis and obtain a solid S



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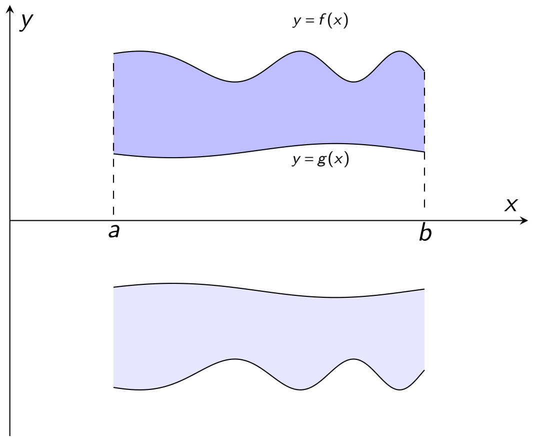

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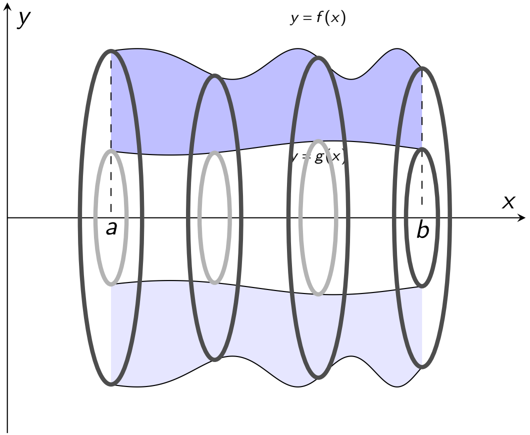
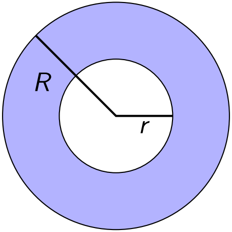


Washer Method

The cross-sections are *annuli*:



If R is the outer radius and r is the inner radius, then the area of this annulus is $A = \pi(R^2 - r^2)$.

For our example, the cross-section at x has outer radius $f(x)$ and inner radius $g(x)$ so

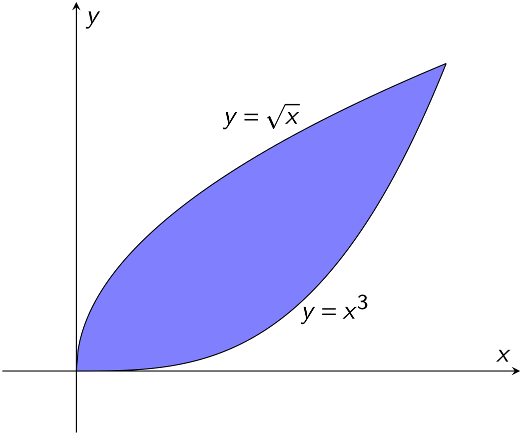
$$A(x) = \pi([f(x)]^2 - [g(x)]^2).$$

Therefore, the volume of S is

$$\text{Vol}(S) = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2\right) dx = \pi \int_a^b (f^2 - g^2).$$

Example 2

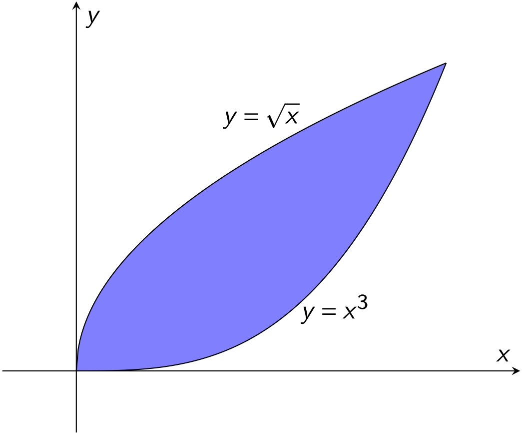
Find the volume of the solid obtained by revolving the region



about the x-axis.

Example 3

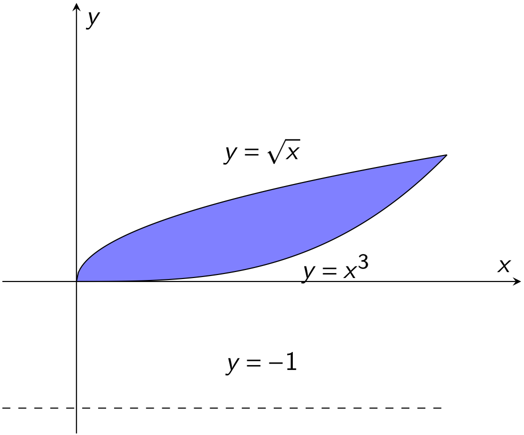
Find the volume of the solid obtained by revolving the region



about the y-axis.

Example 4

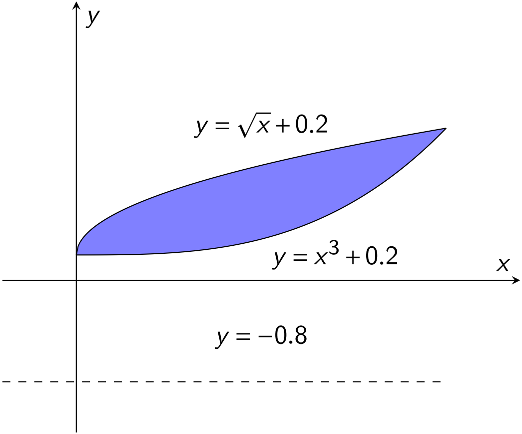
Find the volume of the solid obtained by revolving the region



about the line $y = -1$.

Example 4

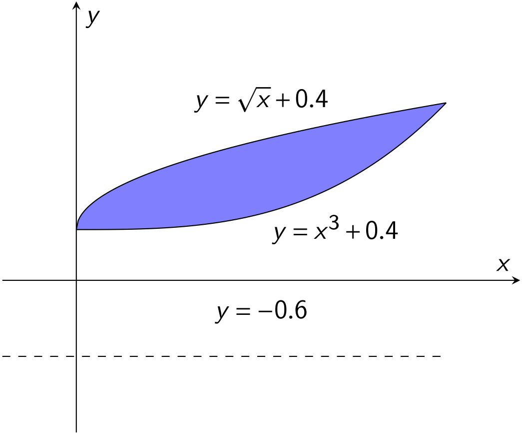
Find the volume of the solid obtained by revolving the region



about the line $y = -0.8$.

Example 4

Find the volume of the solid obtained by revolving the region



about the line $y = -0.6$.

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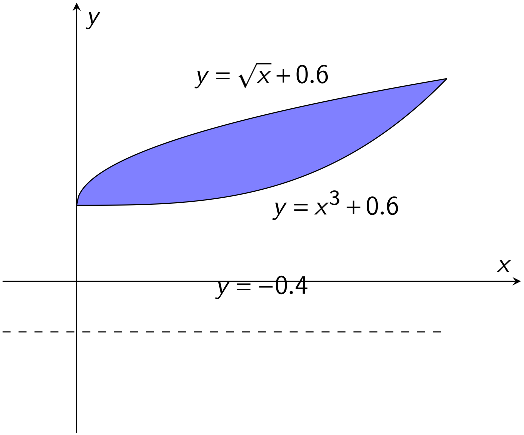
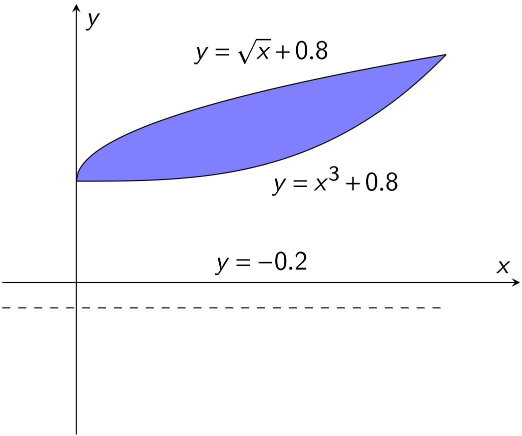
Find the volume of the solid obtained by revolving the region



about the line $y = -0.4$.

Example 4

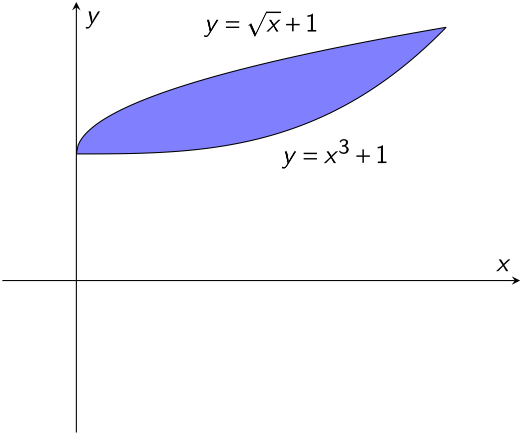
Find the volume of the solid obtained by revolving the region



about the line $y = -0.2$.

Example 4

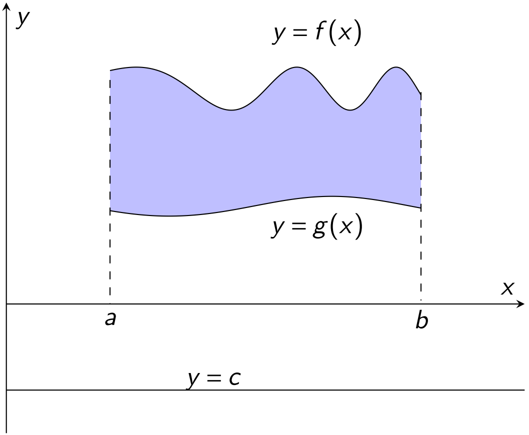
Find the volume of the solid obtained by revolving the region



about the line x-axis.

Other Axes

The volume of the solid S obtained by revolving the region



about the line $y = c$ is

$$\text{Vol}(S) = \pi \int_a^b \left[\left(f(x) - c \right)^2 - \left(g(x) - c \right)^2 \right] dx.$$

Example 5

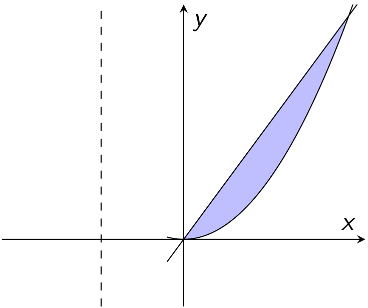
Find the volume of the solid obtained by revolving the region bounded by

$$y = 2x$$

and

$$y = x^2$$

about the vertical line $x = -1$.



Conclusion

If you are rotating around the x-axis, integrate with respect to x.

$$\mathrm{Vol}(S) = \pi \int_a^b [f(x)^2 - g(x)^2]\, dx$$

If you are rotating around the y-axis, integrate with respect to y.

$$\mathrm{Vol}(S) = \pi \int_a^b [f(y)^2 - g(y)^2]\, dx$$