

Homework 6

Due Friday, February 21, 2020

1. Define

$$f(x) = \int_2^x \sqrt{1+t^2} dt.$$

(a) (1 point) Why does f have an inverse?

(b) (1 point) Evaluate $(f^{-1})'(0)$.

2. (2 points) Show that, for all $n \geq 1$,

$$\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} \geq \ln(n+1).$$

3. Evaluate the following integrals.

(a) (2 points) $\int \frac{x}{3-x^2} dx$

(b) (2 points) $\int x \sec(x^2) dx$

(c) (2 points) $\int \frac{\sqrt{x}}{1+x\sqrt{x}} dx$

(d) (2 points) $\int \frac{\sec^2(2x)}{4-\tan(2x)} dx$

(e) (2 points) $\int \frac{e^x}{\sqrt{e^x+1}} dx$

(f) (2 points) $\int \frac{\sin(e^{-2x})}{e^{2x}} dx$

4. (4 points) Define a function $A(x) = \int_1^x \frac{\ln(t) dt}{t^2+1}$. Prove that

$$A\left(\frac{1}{x}\right) = A(x).$$

Hint: Use substitution.