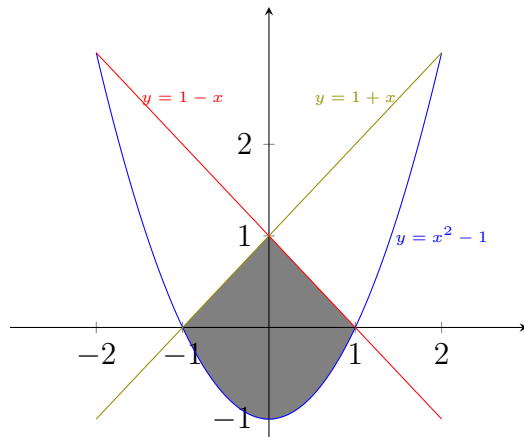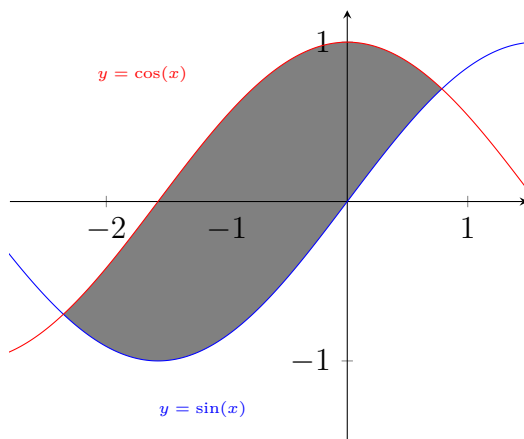# Homework 4

Due Monday, February 10, 2020
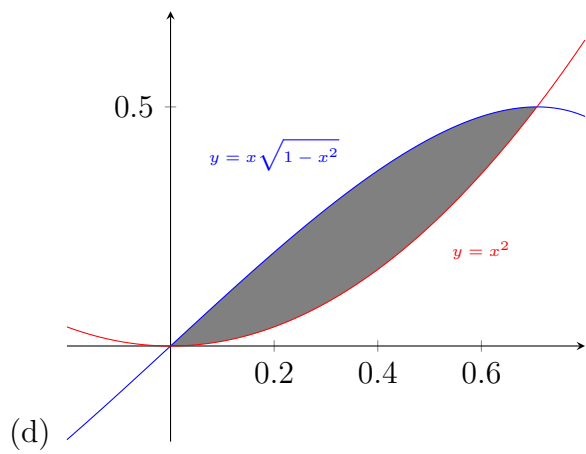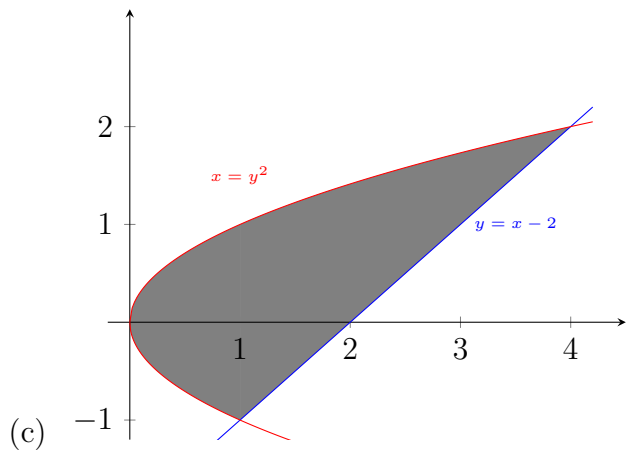
1. (2 points) Find the areas of the following figures.

(a)

(b)

(c)

$x = y^2$

$y = x - 2$

(d)

$y = x\sqrt{1 - x^2}$

$y = x^2$
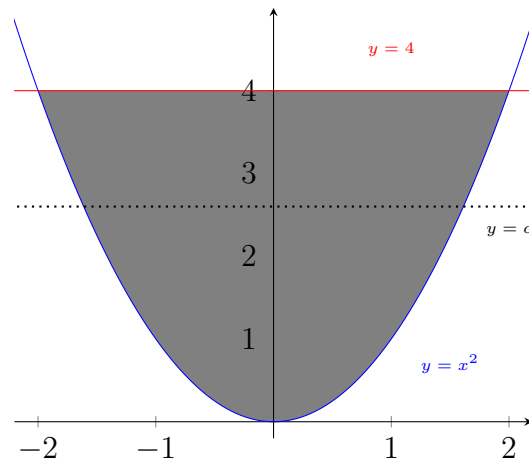
2. Consider the region bounded by $y = x^2$ and $y = 4$. There is a horizontal line $y = c$ which divides this region into two subregions of equal area. Find c.

3. Take $a > 0, b > 0$ and n a positive integer. A rectangle with sides parallel to the coordinate axes has one vertex at the origin and the other vertex on the curve $y = bx^n$ at a point where $x = a$. Calculate the area A of the part of the rectangle that lies below the curve. Show that the ratio of A to the area of the entire rectangle is independent of a and b.

4. The base of a solid is the region bounded by $y = x^2$ and $y = 4$. Find the volume of the solid given that the cross-sections perpendicular to the x-axis are:

   (a) squares with one side on lying on the xy-plane;

   (b) equilateral triangles with one side lying on the xy-plane;

   (c) circles with diameter lying on the xy-plane;

5. Let $0 < h < r$. A sphere of radius r is cut by a plane a distance h from the equator of the sphere. The solid lying above the plane is called a *cap*. Derive the formula for the volume of a cap in terms of h and r.